Online Appendix:

Calculation of Equivalent Variation in Permanent Income Hypotheses Studies

This online appendix explains the calculation of the equivalent variations depicted in Table 1 of the chapter "Natural Experiments in Macroeconomics" for the Handbook of Macroeconomics.

1 Description of Calculations

To calculate the equivalent variation (EV), we assume the following:

- utility is monthly additive
- the household has CRRA utility, $u(y) = \frac{y^{1-\gamma}}{1-\gamma}$, with regular monthly expenditure y and risk aversion $\gamma = 2$
- $\beta = 1$, $r = 0$
- the extra payment is denoted by x; if it is smoothed over 1 year, the extra payment per month is $x/12$

Given these assumptions, the yearly utility from suboptimal behavior is $U^{hand-to-mouth} = 11 \cdot u(y) + u(y+x)$, while the yearly utility from optimal behavior is equal to $U^{rational} = 12 \cdot u(y + \frac{x}{12})$. Denote by z the increase in monthly consumption that would make the household indifferent between the two types of behavior described. To get the equivalent variation (EV), find z that solves

$$\begin{aligned} 11 \cdot u(y+z) + u(y+x+z) &= 12 \cdot u(y + \frac{x}{12}) && (1) \\ 11\frac{(y+z)^{1-\gamma}}{1-\gamma} + \frac{(y+x+z)^{1-\gamma}}{1-\gamma} &= 12\frac{(y+\frac{x}{12})^{1-\gamma}}{1-\gamma} && (2) \\ 11\frac{(y+z)^{-1}}{-1} + \frac{(y+x+z)^{-1}}{-1} &= 12\frac{(y+\frac{x}{12})^{-1}}{-1} && (3) \\ \frac{11}{y+z} + \frac{1}{y+x+z} &= \frac{12}{y+\frac{x}{12}} && (4) \end{aligned}$$

Equation (1) is a quadratic equation in z. It has two roots, and, as will turn out in the examples below, one of the roots will always be negative. We use the positive root when reporting the EV$=z/y$.

Some papers examine the effects of a permanent change in income. In this case, we assume that the change occurs in the middle of the year. x is here the accumulated additional income that accrues in the last 6 months. The formula therefore changes to:

$$6 \cdot u(y+z) + 6 \cdot u(y + x/6 + z) = 12 \cdot u(y + \frac{x}{12}) \tag{5}$$

$$\frac{6}{y+z} + \frac{6}{y + x/6 + z} = \frac{12}{y + \frac{x}{12}} \tag{6}$$

Two different measures of expenditure y can be used to compute the equivalent variation:

1. Total spending or, as second choice, total income reported in each paper; this is used in our baseline approach.

2. A common external measure of total spending; the common external measure which we use comes from Johnson, Parker, Souleles (AER, 2006), which give average quarterly household spending as 4149 in 2001 US Dollars; we multiply by 4 and use CPI data to bring this amount to current dollars in the year of the respective experiment.

The advantage of the first approach is that it realistically captures the expenditure of the households in the study, which are often not a representative sample of the total population, but rather a sample of a specific subpopulation (for example, home owners, or social security recipients). The disadvantage of this approach is that the measures might not be directly comparable, since they are in some instances referring to expenditures, in some instances to income. The second approach uses a common external measure of total spending for all, which might however not be a good proxy for spending of the specific subgroup analyzed in each study.

In what follows, we describe which exact values are used to calculate the equivalent variations for each paper.

2 Using Data on Total Spending from Each Paper

2.1 Temporary Change in Income

1. Agarwal, Liu, and Souleles (JPE, 2007)

   Plug in Equation (4) the following values for x and y:[1]
   $x = 300$ (page 1, for singles)
   $y = 327 * 5$ (Table 1, average monthly spending on credit card)
   327 is only the amount of spending done via credit card, but x is the total government transfer. The authors cite Chimerine 1997 at page 988 to indicate that approximately 20 percent of total consumption is paid by credit cards. We use y=327*5 as approximation.

2. Agarwal and Qian (AER, 2014)

   Plug in Equation (4) the following values for x and y:
   $x = 511$ (Table 1, Panel A, average monthly benefit for matched treatment group)
   $y = 6644$ (Table 1, Panel A, average monthly income for matched treatment group in 2010 Dollars)

3. Broda and Parker (JME, 2014)

   Plug in Equation (4) the following values for x and y:
   $x = 898$ (Table 2, average tax rebate if *rebate* >0)
   $y = 179 * 30/7 * 100/19$ (Table 2, average weekly spending if *spending* >0; we multiply the weekly spending by $\frac{30}{7}$ to get the monthly equivalent)
   Broda and Parker (2014) use data from the Nielsen Consumer Panel, which captures a limited share of consumption goods. Since we want to use weekly spending as measure for total consumption, we scale y by 100/19. This approach follows the authors' assessment of the coverage of total consumption provided by their data:
   'But to put this issue in perspective, (weighted) spending per capita in the NCP is about \$ 57/week which is about 10 percent of NIPA per capital PCE. At the household level, spending is 35 percent of spending on broad nondurable goods reported in the 2008 CE Survey or 19 percent of total consumption spending.' (Broda and Parker (2014): p. 25)

[1] In parenthesis, we list the page/table number in the each paper from which the values for x and y are drawn.

4. Browning and Collado (AER, 2001)

   Values for x and y cited below are in Spanish pesetas. To compute the EV for this paper, we modify the expression in Equation (1) as follows:

$$10u(y+z) + 2u(y+x+z) = 12u(y+\frac{2x}{12})$$

   where:
   $x = 408616 * 4/14$ (Table A2, bonus size is 1/14 of annual earnings (quarterly earnings*4). Bonus is paid twice a year. Therefore, we have 10 months where the household receives 1/14 of annual income and 2 months where 1/7 of annual income is disbursed)
   $y = 668022/3$ (Table A2, total quarterly expenditures).
   The EV in this case should make the household indifferent between smoothing consumption over all months within a year (12 months) and consuming more in the months when bonuses are received (2 months).

5. Hsieh (AER, 2003)

   Plug in Equation (4) the following values for x and y:
   $x = 2048$ (Table 1, Alaska bonus, 1982-1984 US-Dollar)
   $y = ((713 + 1107) + (643 + 1109))/2$ (Table 1, average monthly spending on durables and nondurables over two periods (first parentheses: Jul-Sep; second parentheses: Oct-Dec); i.e. author provides average numbers over each 3 months period, and we average again over both periods)

6. Johnson, Parker, and Souleles (AER, 2006)

   Plug in Equation (4) the following values for x and y:
   $x = 480$ (Table 1, p. 1594, $taxrebate|taxrebate > 0$)
   $y = 47021/12$ (Table 1, p. 1594 total annual income)

7. Parker, Souleles, Johnson, and McClelland (AER, 2013)

   Plug in Equation (4) the following values for x and y:
   $x = 970.8$ (Table 6, average tax rebate conditional on $rebate > 0$)
   $y = 10601/3$ (Table 6, average quarterly spending on goods and services)

8. Souleles (AER, 1999)

   Plug in Equation (4) the following values for x and y:
   $x = 874$ (Mean average real refund for households in CEX data, 1982-84 US Dollars (Table 1, p. 949))
   $y = 21522/12$ (Real gross annual household annual earnings (1983 Dollar) from Souleles (2002) (Table 1, p. 105))

2.2 Permanent Change in Income

1. Aaronson, Agarwal, French (AER, 2012)

   Plug in Equation (6) the following values for x and y:
   $x = 237 * 2$ (The permanent minimum wage increase yields 237 Dollar more income per quarter (p. 3116) per household with fraction of minimum wage income higher than 20 percent. It is multiplied by 2 since at this point we assume half a year of increased income for our analysis. x is in 2005 Dollars.)
   $y = 6462/3$ (Average real quarterly spending, 2005 Dollars (Table 2, p. 3119))

2. Coulibaly and Li (REStat, 2006)

   Plug in Equation (6) the following values for x and y:
   $x = 277 * 6$ (Table 1, average mortgage payment for payoff sample, 1982-84 Dollar for half a year)
   $y = 1785$ (Table 1, average total consumption for payoff sample)

3. Parker (AER, 1999)

   (a) The first experiment analyzes a permanent change in the social security tax rate, which we assume applies in the middle of the year. The paper lists 6 episodes of increases of social security tax rates.
   Plug in Equation (6) the following values for x and y:
   x is in this case the average increase in individual tax rate [End rate-Initial rate in each episode/number of episodes] times the full sample monthly pre-tax family income of 2241 Dollar from Table 2 times 6 for half a year:
   x=(0.0052+0.005+0.0035+0.001+0.0036+0.0014)/6*2241*6=34.06 Dollar.
   y=1449 (The average monthly total expenditures of a household (Table 2, p. 964)).

(b) The second experiment analyzes the increase in household income when a household reaches the annual social security cap. We assume that for the last three months of the year, the household does not pay social security taxes anymore. The following formula is used:

$$9 \cdot u(y+z) + 3 \cdot u(y + x/3 + z) = 12 \cdot u(y + \frac{x}{12}) \tag{7}$$

$$\frac{9}{y+z} + \frac{3}{y + x/3 + z} = \frac{12}{y + \frac{x}{12}} \tag{8}$$

x=330*3 (The temporary monthly increase in income from October until December (p. 959)).
y=1449 (The average monthly total expenditures of a household (Table 2, p. 964)).

4. Scholnick (REStat, 2013)

   Plug in Equation (6) the following values for x and y (in Canadian dollars):
   $x = 751.46 * 6$ (Table 1, p. 1445, average final monthly mortgage payment)
   $y = 64554.90/12$ (Online Appendix Table 1: average income of treatment group families)

5. Shea (AER, 1995)

   Plug in Equation (6) the following values for x and y :
   $x = 27960 * 0.006/2$ (Table 2, p. 192, average annual household income*expected average yearly real wage growth rate (EDWAGE), divided by 2 to give increase in income under assumption that change occurs in middle of year)
   $y = 27960/12$ (Table 2, p. 192 average annual household income deflated to 1982 US-Dollars)

6. Souleles (JPE, 2002)

   Plug in Equation (6) the following values for x and y:
   $x = 117 * 2$ (Average change of quarterly withholding using the WHOLDP measure (p. 106)*2)
   $y = 21522/12$ (Real gross household earnings (1983 Dollar) (Table 1, p. 105))

7. Stephens (REStat, 2008)

   Plug in Equation (6) the following values for x and y:
   $x = 406 * 6$ (Table 1, monthly vehicle payments for consumer units paying off loan, 2000 Dollars, times 6 for half a year)
   $y = 39900/12$ (Table 1, average annual after tax income for consumer units paying off loan)

3 Using External CPI-Adjusted Data on Total Spending

To calculate the EV based on CPI-adjusted spending of households, we use the value for y (average quarterly spending in 2001 = \$ 4149) from Johnson, Parker and Souleles (AER, 2006), multiply it by 4 to get annual spending, and adjust it to the respective year using the CPI data from `http://www.minneapolisfed.org/community_education/teacher/calc/hist1913.cfm`.
The adjustment is done in the following manner:

$$y_t = y_{2001} \cdot \frac{CPI_t}{CPI_{2001}}$$

In the cases where the experiment in the paper covers multiple years, adjustment of y_t is done based on the midyear of the data sample (t in the equation above is set to the midyear). For singular events, y_t is adjusted to the year when the event occurred (e.g. $t = 2008$ for papers on the 2008 Stimulus Payments). No results are reported for papers that have experiments in different currencies (e.g. Stephens and Unayama, 2011, who use data expressed in Japanese yen). The values for x (the extra payment) and the equation used to calculate the EV are the ones described earlier, except for the following two cases:

3.1 Temporary Change in Income

- Agarwal, Liu, and Souleles (JPE, 2007)

  Use x on household level, rather than on individual level:
  $x = 500$ (500 Dollar for households, p. 987, 2001 Dollars)

3.2 Permanent Change in Income

- Souleles (2000)

  Value on y not given in paper, therefore not part of previous analysis:
  $x = -980 * 2$ (p. 191, average household real college expenditure in quarters where it

is positive. Data in 82-84 US Dollars. Households expect their children to enter college in the middle of the year and to be obliged to pay twice quarterly college expenditures. Therefore, we multiply the quarterly value by two.)

4 No Equivalent Variations are Calculated for the Following Papers

All papers relying on payment schedules of positive payments on a single day per month and zero payments in between are not included into the EV calculations (not smoothing in the spirit of our exercise would imply zero consumption in between payment days). These are the following papers:

1. Mastrobuoni and Weinberg (2009)
2. Shapiro (2005)
3. Stephens (2006)
4. Stephens (2003)

Moreover, the following papers are not included in the EV calucations:

1. Paxson (1993): This paper relies on comparing seasonal spending patterns of farmers and non-farmers in Thailand, who have different seasonal income patterns; therefore, there is no specific extra payment analyzed.
2. Stephens and Unayama (2011): This paper analyzes a change in the frequency of Japanese Public Pension Payments, going from payments every three months to payments every two months; thus, there are zero payments in between, just as in the papers above relying on daily payment schedules.
3. Wilcox (1989): This paper does not provide enough information on the sizes of the social security amount increases in order to calculate the extra payment.

5 Results

The following table gives for each paper the calculated equivalent variation for both spending measures.

Paper	**EV (total spending from paper)**	**EV (external total spending)**
Aaronson, Agarwal, and French (2012)	0.03%	0.11%
Agarwal, Liu, and Souleles (2007)	0.22%	0.75%
Agarwal and Qian (2014)	0.04%*	-
Broda and Parker (2014)	0.31%	1.45%
Browning and Collado (2001)	2.61%	-
Coulibaly and Li (2006)	0.56%	2.63%
Hsieh (2003)	4.79%	14.87%
Johnson, Parker, and Souleles (2006)	0.10%*	0.69%
Parker (1999)[1]	0.00038 %	0.00110%
Parker (1999)[2]	0.82%	2.13%
Parker, Souleles, Johnson, and McClelland (2013)	0.46%	1.65%
Paxson (1993)	-	-
Scholnick (2013)	0.45%*	-
Shea (1995)	0.0009%*	0.0085%
Souleles (2002)	0.01%*	0.06%
Souleles (2000)	-	5.24%
Souleles (1999)	1.24%*	4.65%
Stephens (2008)	0.35%*	1.95%

Notes: Papers written with an asterisk(*) use an income measure rather than a spending measures as basis for the expenditure measure y in the calculations.

1 Change in social security tax rate

2 Cap in social security withholding